THEOREM: THERE EXISTS A GOD

MATHEMATICIAN'S ONOTOLOGICAL CONVERSATIONS WITH GOD

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Abstract. We study the content of several ontological arguments towards the existence of God provided by notable scholars during the years *c*.1000 − 2000. We show that mathematical and theological responses to the ontological arguments parallel the move towards secularization in Europe, and that the socio–political and academic environment emerging from Romantic era Germany influenced a philosophical inversion between traditional theological and traditional rationalist tenants concerning "Man's" capacity to comprehend God, the divine, and the eternal. We apply our analysis to explain why despire Leibniz and Cantor being virtually indistinguishable as individuals, Cantor was lambasted by his contemporaries whereas Leibniz was universally praised.

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1. Introduction

In the 11^{th} century a Catholic Bishop in the Holy Roman Empire by the name Anselm of Canterbury attempted to craft a formal argument for the existence of God. "Anselm's ontological argument," as it is known, is the first documented instance (post–antiquity) of a rigorous attempt at such a formalized proof. This work holds significance in the larger context of theological discussions in post–dark age Europe; in particular as it pertains to axiomatizing religious foundations and the divine. Gaunilo and Aquinas's objections and counter–arguments levied against Anselms "proof" would supress theological and philosophical debates over the potential redeeming value of Anselm's argument for five–hundred years. Only with Descarte's reformulation of Anslem's ontological proof does one see a reinvigoration of topic of proving God's existence. Inspired by the Descartes and Spinoza, Leibniz presented his own proofs that argue towards the existence of God. The protestant reformation and the scientific revolution have a particularly noteable affect on these existence arguments: Leibniz's arguments are characteristically more mathematical and embedded in axioms that are, in a sense, independent of human experience. In contrast, Anslem's original proof relied on the cognitive ability of Man[1]. We likewise observe a similar phenomena in the works of Cantor, whose (successful) attempts to mathematically understand the notion of infinity would cause him to chastized by his mathematical contempories and nearly go insane. Cantor believed one could more successfully understand

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[1]Uppercase to be synonymous with "human-kind"

God through a rigorous mathematical study of the different sized of infinity. This philosophy, in conjunction with his faltered reputation in the mathematical community, would lead Cantor to spend the rest of his life collaborating with theologians within the Cathloic clergy, including Pope Leo XIII. Cantor hoped if the Church was going to define God using infinity, then they should at least have a correct mathematical foundation. The resolution to this lineage on ontological proofs may be realized in Kurt Gödel at the beginning of the 20^{th}–century, whose *Incompleteness Theorem* has often be used by theologians as another "proof" of God's existence. The universal mathematician died out with Hilbert, Gödel's contemporary. Since then, and partially by the influence of Bourbaki, there has been no serious effort by professional mathematicians to prove God's existence.

1.1. **Warning to the reader.** No single paper can treat all interactions between mathematicians and Christian theologians during the post–dark age world. Indeed it is even difficult to give a satisfactory treatmeant of all mathematical–philosophical thought occuring during the time–periods considered in the present investigation. It is for this reason that we focus on the theological and mathematical motives for constructing an argument towards the existence of a supreme being. If we further restrict attention to the Christian world, then we naturally are led to the four historical figures mentioned above. We will show that Anselm, Descartes, Leibniz, and Cantor form a coherent lineage of pseudo–mathematical philosophers concerned with proving the existence of God, each being influenced by the prior. Moreover, any philosophical debates concerning a quasi–rigorous "proof of God" occuring between $c.1000 - 2000$ can be directly traced back to at least one of the figures above (hence all that came before him as well).

We also note that the distinction between pre and post–reformation philosopy is echoed strongly in the formal structure of our philosophers' arguments. In paritcular, Leibniz's and Cantor's considerations are more grounded in precise mathematics than that of Anslem and Descartes. This is no suprise given that the former were trained mathematicians. However, it is also important to note that Leibniz (ergo Cantor) was heavily influenced by influx of intellectualism in post–reformation Germany, whereas Descartes is the epitome of revolutionary and anti-status-quo thinkers. Given these distinctions, we focus primarily on Leibniz and Cantor's work, but will introduce Anselm and Descartes' influence for completion and context. For more details about Descartes' influence on mathematics, see [30].

2. Preliminaries: the ontological argument

2.1. **Anselm vs. Aquinas.** In this section we describe Anselm's ontological argument, initially presented in his *Proslogion* [31]. The reductio ad absurdum proceeds by making three fundamental assumptions and one definition, and then logically showing that the two *Anselian* assumptions are incompatible. The first assumption that Anselm makes is that the human experience is necessary, i.e that people are meant to be on Earth through some cosmic or divine reasons. This supposition leads to circular reasoning, but it is not the main point of contention for Anselm's contemporaries, nor for Thomas Aquinas. Additionally, Anslem was not aware that this was an optional hypothesis in his argument, a characteristic not present within future ontological proofs, given mainly by Leibniz. Although he was cognizant of the need to define God:

Definition 1 (Anselm [31])**.** "[God] is a being than which nothing greater can be concieved"

He goes on to hypothesize that "the fool who says in his heart that there is no God" [Psalms 14:1]

- understands the notion of God
- but does not believe such a God exists,

Claim 1. *There exists a God*

Proof. "'... And assuredly that, than which nothing greater can be conceived, cannot exist in the understanding alone. For suppose it exists in the understanding alone: then it can be conceived to exist in reality; which is greater. Therefore, if that, than which nothing greater can be conceived, exists in the understanding alone, the very being, than which nothing greater can be conceived, is one, than which a greater can be conceived. But obviously this is impossible. Hence, there is no doubt that there exists a being, than which nothing greater can be conceived, and it exists both in the understanding and in reality...." [31] □

We remark that Anselm's definition is the main point of contention against his arguments. Furthermore, this contention is what would eventually inspire Descartes to make an attempt at the argument that is independent of the definition of God. The most influential counter–argument to Anselm provided by one of his contemporaries is that Gaunilo of Marmoutiers, an influential French monk. In his work "On behalf of the fool" [32], Gaunilo articulates that if Anselm's argument is correct, than one could similarly prove the existence of "the greatest concievable Island" (in a time when many of the world's land-masses had still not been discovered by Europeans). Since a proof towards the existence of the greatest conceivable island is logically inconsistent until Man has charted all islands, Anselms proof cannot be correct. However, the most suffering blow to Anselm's ontological argument came over one-hundred years after the Anselm's death in the works on Saint Thomas Aquinas. Cosgrove points out that "[Aquinas] criticizes Anselm's argument for God's existence in five works: *In Primum Librum Sententiarum, In Boethii De Trinitate prooem, Quaestiones Disputatae De Veriate, Summa Contra Gentiles,* and *Summa Theologie*, with the latter being the best known and most representative of Aquinas's response [to Anselm]" [9]. In it, Aquinas rejects that "one can reduce God to a singular definition, as not everyone shares the same concept of God." While this point of disagreement is easily reconciled, it would be five-hundred years before any philosopher pushed back against Aquinas's dismissal of Anselm's efforts. Descartes was the first. Aquinas gave a seemingly sound logical argument agains the supposition that everyone's interpretation of God is identical: "if so, then is does not follow that [noone] understands what the word [God] and the notion [of God] actually signifies, but only that it exists mentally." Notably, Aquinas also criticizes Anselm's attempt to "understand that which is beyound all conceivability". Kenneth Himma observes that "on this view [in Aquinian philosophy], God is unlike any other reality known... we can easily understand concepts of finite things, the concept of an infinitely great being dwarfs human finite understanding" [19]. This philosophy of *the finite Man* is fundamental to Christian philosophy for almost all of pre–reformation Europe. Not simply due to the influence of Aquinas, but the supreme power of the Holy Roman Empire was notoriously unamicable towards heretical claims to understand more than the finite realm gifted by God[2]. This notable feature of Thomism, and Catholicism in general, is characteristic of theological opposition towards any notion of being able to prove the existence of God. This philosophical conservatism is not reconciled until the rise of the neo-Thomists and neologians in the 19^{th} century; however, Positivism in the sciences converts the academic grandchildren of the rationalists into the primary purveyors of the finite Man.

Aquinas "believed that God's existence is self–evident, and rejected the notion that existence could be deduced from claimed properties about God," i.e Aquinas was adamantly against axiomitzation of the concept or physical repesentation of God [19]. This is in contrast to the utilization of symbolic abstraction by many philosophers after Descartes, especially mathematicians [30]. Nevertheless, the Church's central objection to any formulation of a proof for God's existence would continue to be seen as contrary to Man's place on Earth up through Leibniz's time. Only after the Romantic era, and in the works of Schleiermacher and Fries do we generally observe a shift in attitude, both in theologians and mathematicians.

2.2. **Descartes and pre-axiomitization.** The criticisms levied against Anselm by Aquinas in particular disuaded scholarly discussions on the possibility to prove God's existence for hundreds of years [9]. Descartes, to the surpise of some of his contemporaries, reinvigorated the study of such considerations [28]. In his *Third and Fifth Meditations*, he started with a definition of God compatible with that of Aquinas:

Definition 2 (Descartes [6])**.** God is an infinitely perfect being.

Descartes actually gave a few proofs for the existence of God, all of which he claimed to be equivalent, but gave no proof of this claim. Moreover, Descartes would often try to trivialize the existence of God, comparing it to "the obvious fact that a triangle cannot have two right angles." Advocacy for a tautological argument towards God's existence is supported by many, including Aquinas and Schleiermacher. We focus on what is known as "Descartes's ontological proof," which follows in the spirit of Anselm's. Descartes's "causal proof" is described in [4].

Claim 2. *There exists a God*

Proof. By definition, God is the infinite perfect being. An infinitely perfect being must have existence, since otherwise it would not be infinitely perfect. Hence God exists. □

[2]This claim is an assumption we take for granted. It is supported by the content of class lectures, and well–known facts about the Church's persecution of enlightenment figures.

Descartes subtly assumes that existence is a quality of perfection, but does not give proof. Leibniz's arguments for the existence of God also utilize the notion of perfection, but he is vastly more aware of subtle details that need additional argumentation than Descartes or Anselm [35]. Leibniz takes care to argue that existence is a quality of perfection, "comparing nonexistence vs. existence to the binary system $\{0, 1\}$, with 0 being nil-quantity and 1 possessing quantity, ergo is [relatively] perfect." (Leibniz's heavily referenced the binary system to justify theological arguments [20]). One sees glimpses of Leibniz's binary philosphy in the transfinitism[3] of Cantor. Additionally, the defintions (resp. claims) 1 and 2 (resp. 1 and 2) have neglected to address the uniqueness problem of God, i.e is there a singular God[4]. Leibniz is the first documented mathematical philosopher to address this issue as well [35].

Ignoring criticisms of Descartes simple proof, the most prominant and referenced objection to Descartes's argument comes from his contemporary Johannes Caterus, who levies effectively the same charge agains Descartes as Gaunilo and Aquinas did against Anselm:

> ...Even if it is granted that a supremely perfect being carries the implication of existence in virtue of its very title, it still does not follow that the existence in question is anything actual in the real world; all that follows is that the concept of existence is inseparably linked to the concept of a supreme being. So you cannot infer that the existence of God is anything actual unless you suppose that the supreme being actually exists; for then it will actually contain all perfections, including the perfection of real existence [17]

In Caterus's refutation we obseve the pervasiveness of pre–reformation theological tenants concnering the finiteness of human understanding. Thomas Hobbes likewise lambasted Descartes, writing "the things that are said in this Meditation make it clear enough that there is no criterion by which we can distinguish our dreams from the waking state and from truthful sensations." And Pierre Gassendi notes "there being human nature at a time when there are no human beings, or of the roses being a flower (that great eternal truth!) at a time when not even one rose exists, also attempting to point out to Descartes the fundamental finiteness conditions that *all life on Earth* are subject to, whilst God is eternal and unbounded [14]. In particular, not only is Descartes's ontological proof morally indistinguishable from Anselms, but there was not significant difference in the theological reception of their arguments. As noted in the introduction, it is for this reason and the reason's highlighted in the previous paragraph, that the transition from Leibniz to Cantor's mathematical–theological reports are more interesting and more dramatic.

3. Leibniz's Essays

"No matter which part of Lebniz's thought one begins with, one soon reaches God" [35]. Leibniz was born into an academic family at the height of post–reformation Euphoria in Germany [3, 1]. The socio–political significance of his birthyear and birthplace cannot be underestimated. While influenced by Descartes [30], in the realm of theology "Leibniz attempted to fill what he saw as a shortcoming in Descartes' ontological argument" [6]. For him, "[Descartes'] proof is valid only if one first proves the existence of an infinitely perfect being" [35, Ch. 2.1]. This consistency is also characteristic of Cantor's mathematics, and is one of the influential factors in our reconstruction of what we claim to be "Cantor's lost ontological proof" [section 4.2]. Leibniz adopts an interesting strategy in his original ontological proof. He retains the Thomist defintion of God, but originally defines God as "a being from itself, i.e a being from whose essence existence follows, i.e a necessary being." In a letter documenting a face-to-face conversation with Huthmann, Leibniz writes:

> ... if a necessary being is possible, then it actually exists...

Leibniz proceeds to recount his proof of the argument. It follows that if God is possible, then he will actually exist (using the definition as a necessary being). Leibniz abstractly argues that "the possible existence of God and his actual existence are inseparable" [35, 2.4(4)]. To bridge the gap with Thomist philosophy, Leibniz is sure to also prove "the essence of God and supreme perfection are inseparable" [35, 2.4(NB 1)], and similarly the essense and existence of God are inseparable. Leibniz doubles down on his mathematical pedanticism and proves that "the existence of *a* God implies the existence of *the* unique God" [35, 2.2]. This attention to logical detail distinguishes Leibniz from his predecessors. Summarizing Leibniz's stance,

[3]See section 4 for definitions

[4]And so we have taken care to use the descriptive *a* instead of *the*...

Definition 3 (Leibniz [35])**.** God is *the* being having supreme perfection (equivalent to God being a *necessary* being).

Claim 3. *There exists a unique God.*

Proof. If a necessary being is possible, then it exists; existence, essense, and supreme perfection of a necessary being are inseparable; God is necessary and possible, hence God exists. □

Leibniz enjoyed a particularly unique elevated status during his lifetime. His academic works are numerous and transcend disciplines, often intimidating his contemporaries [22]. His position in the burgeoning post–reformation academic climate of Germany appears to have shielded Leibniz from acadmic scrutiny within Lutheran Germany. Moreover, Leibniz's attempts at the reunification of the churches protected him from any serious Catholic criticism, with "Pope Innocent IX commending Leibniz for his scholarly efforts" [35, Ch. 3]. Nevertheless, it remains unclear to the author how much of Leibniz's more ambitious theological projects, especially those involving the Catholic church, were done out of legitimate interest, or if Leibniz was simply playing politics. Despite Pope Innocent's appraisal, he has also said that "[Leibniz] did not fully understand the tenants of faith" [20]. Moreover, Leibniz himself was not shy of offering harsh criticism of ecclesiastical institutions. In his *Dialogue between a theologian and a misosophist*, Leibniz writes:

> ... For when they [people who use blind faith] introduced serious problems they pretended that they were yielding to the authority of the Church and that these problems were not obstacles. There is no greater enemy of religion and piety than he who asserts faith contrary to reason, which is to prositute faith before the wise...

In any case, what we can definitiely gather from these essays is that the liberalization of the Catholic church had begun in Leibniz's time, but the Catholic authority was still not prepared to compromise on the finite nature of Man and his understanding of the divine. This is only different than the religious dismissal of Anselm and Descartes' arguments in that Leibniz's super–star status, both amongst his contemporaries and within academic communities for the remainder of known history, escaped from scrutiny almost entirely unscathed. The same is not true for Cantor in the following century, despite the fact that Leibniz and Cantor were both German–born Lutheran mathematical–philosophers who lived in a post–reformation, post–scientific revolution era and believed that mathematics was a divine purpose. Our central precise is that the distinction lies in philosophical involution between theology and science that occured during the era following Leibniz; namely, the Romantic era. Romantic science, developed by Schelling, would form philosophical foundations for Catholic neo–Thomism and Protestant neologism in Europe. This in turn would spark reactions in the rationalism and scientific communities which would turn out unfavorable for Cantor's mathematical ambitions.

4. Cantor's Transfinitism

4.1. **The Romantic Era and Profinitism.** In order to understand the differences in reactions by contemporaries to Leibniz's and Cantor's work, it is important to take into account the role that Romanticism played in re-shaping natural–philosophical (i.e scientific) interpretations and attitudes towards God. Friedrich Schelling's thoeries, collectively referred to as *Naturphilosophie*, heavily influenced Romantic science [10]. Amongst his followers is the Kantian theologian Friedrich Schleiermacher, whose *Speaches on Religion* (1799) "called for a reconciliation between Christian theologians and rataionalist natural philosophers," while simultaneously attempting to redefine religion [10, pp 72]. Implicit in Schleiermacher's definition of religion as "that component of human consciousness that was neither cognitive nor volitional" is a reverberation of Schelling's description of God as "the result of transporting the absolute spirit [in Man] to the infinite end of time" [ch. 1]. Indeed Schleiermacher often refers to God as "the infinite [being]," and advocated, like many Kantians, that Man could understand God given a (countably)[5] infinite amount of time. We will refer to this particular theological philosophy as *Profinitism*. Schelling's (hence Schleiermacher's) theories would not withstand the transition into Positism (c. 1840), and Scientists of the mid-to-late 19^{th} century would often criticize Schelling's *Naturphilosophie* as being too unconcered with empirical evidence [2]. The socio–academic atomosphere in the immediate aftermath of Romanticism would not help Cantor's mathematical pursuits. The hyper–reactionary transition meant that most of Cantor's contemporaries would ridicule

[5] This distinction will become important to understand the difference between this and Cantorian mathematical theology

his works as being either too theological and/or too philosophical, with Kronecker[6] writing "I don't know what predominates in Cantor's theory philosophy or theology, but I am sure that there is no mathematics there" [5].

Nevertheless, Schelling and Scheielermacher's works were signifiant in that they inspired theological counter–movements which would survive in tandem with the 19^{th} century scientists. In particular the so-called "neologists" and the philosophical works of Jakob Fries are born out of response to Schelling's followers. Jakob Fries' 1805 (resp. 1808) work *Knowledge, Belief, and Aesthetic* (resp. *New Critique of Reason*) stems from disagreements with Scheiermacher's notion of profinitism, arguing "feelings of the beauty and sublimity of nature we encounter the eternal in the *finite*" [pp. 75]. The notion of the finiteness of Mann is strongly echoed in the outlook of scientists of the post–rationalist era, in particular with the rapid secularization induced "by Marxism, the explosion of agnostic and atheistic social-circles, the findings of Darwin, and the rise of anti-clericism across Europe" [7]. The neologists, whose philosophy emerged in parallel and ultimately merged with that of Fries, were the pioneers of the "theological rationalism [movement], which would become immensely popular in the 19^{th} century" [10, pp.71]. These theological rationalists arose in tandem with Catholic and German piestics theologians, "unwilling to argue that revelation had to be combatable with reason" and focusing on grandiose problems, such as arguments for the existence of God. It would ultimately be religious scholars within this doctrine of theology that would open up to Gregor Cantor in the late 1800's, including the German Catholic rationalistic theologian Constantine Gutberlet [29].

Ironically, where Leibniz failed to reunify the churches, pietism revivals in Protestantism together with a modernization of the Cathloic church would ultimately lead to a quasi–unificaiton of Catholic and pieist Protestant theology by the end of the 19^{th} century. However, factors including Voltaire's criticisms of the Catholic Church, the French Revolution (1798–1815) and the subsequent transition of power from church to state, and papal reformations instantiated by Pope Pius IX and the Vatican council between 1860 and 1870, including the affirmation of papal infallibility, and Otto von Bismark's *Kulturkampf* (1873) against Catholics would initially cause deep strife between the German protestants and the Catholic church, hindering Cantor's ability to gain reputibility in Germany. This ultimately results in Cantor's theological collaborations with Catholic Theologians, including Pope Leo XIII, who was also responsible for the peace negotiations with Otto von Bismarck [8]. These normalized relations between the powers, and Cantor was ultimately able to continue correspondence with his German mathematical contemporaries, including Dedekind and Mittag-Leffler [13]. But the period 1870-1880 was marked by deep psychological paranoia for Cantor, as he continuously postponed publishing his papers on set theory in fear of heavy criticism, most notably from Kronecker and Poincaré [12]. The former once reffered to cantor as a "scientific charlatan," and the latter called Cantor's theories "a grave disease [to mathematics]" [13]. Moreover, Cantor's reputation further suffered following 1874 when he neglected to reference Dedekind in his proof that the rational numbers are countable [26][7]. This gap in mathematical activity is filled in by his theological collaborations with Neo–Thomist catholics Tilman Pesch, Joseph Hontheim, and Gutberlet [11].

4.2. **Cantor's proof.** By *Transfinitism*, we shall refer to the acceptance of Cantor's transfinite numbers [see below] in contexts outside of pure mathematical thought, specifically as it pertains to the assertion that Man's ability to comprehend God is beyond a finite capacity.

Cantor is known for his work in set–theory. Specifically in his formulation of transfinite numbers, known as *ordinals* and *cardinals*. Before proceeding, we give a very elementary overview of these ideas: for additional details, see [33]. Rougly speaking, an *ordinal* is a set $\mathcal{O}$ consisting of objects that can be put into a coherent order. Examples include the natural numbers $\mathcal{O} = \{1, 2, 3, \dots\}$ since we know from human experience that 1 is smaller than 2, which is smaller than 3, etc. A nonexample is when $\mathcal{O}$ is the family tree of a person P with a lot of cousins: while it is possible to make sense of "$P <$ father of P", it is not logical to say "P ¡ cousin of P" or vice–versa. In general, if we can compare the relative "sizes" of any pair of elements in $\mathcal{O}$, then the set forms an *ordinal*. Similar, yet mathematically and philosophically distinct, is the notion of a *cardinal*, which is just a classification of the size of a set, regardless of whether-or-not its members can be ordered. So while $\mathcal{O}$ in the family tree example is not an ordinal, it is a cardinal. In fact it defines the *same* cardinal as the the set $\{0, 1, \dots, N\}$, where N is a very large finite number, since it is possible to define a

[6]A massively influential number Theorist

[7]Every scientific community can be harsh

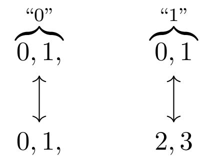

FIGURE 1. Picture showing that $\{0,1\}^{\{0,1\}}$ is the same ordinal as $\{0,1,2,3\}$.

$1-1$ correspondence between $\mathcal{O}$ and $\{0,1,2,\ldots,N\}$, e.g 0 corresponds to P, 1 corresponds to "father of P", 2 to "mother of P", etc[8]. Importantly, Cantor's math says that the cardinal defined by the natural numbers, usually denoted $\aleph_0$, is strictly smaller than the ordinal defined by the real numbers, denoted $\aleph_1$. Similarly, the smallest countably infinite ordinal ω[9] is strictly smaller than the ordinal obtained by repeating ω an ω–number of times, denoted ω^ω. One can think of this process as being analogous to writing down the two numbers 0 and 1 on a page from left to right, and then repeating the two numbers once more in a line. We call the first pair the 0^{th} pair, and the second is the 1^{st} pair. We can then relabel the numbers (using the left-to-right ordering) as $0,1,2,3$. This fact, which clearly has philosophical underpinnings, is phrased in Cantor's mathematics as "the set $\{0,1\}^{\{0,1\}}$ has the same ordinality as $\{0,1,2,3\}$:" see figure 1. For finite sets, cardinals and ordinals are the same. Cantor's genius is that he formalized a rigorous mathematical system of handling infinite ordinals and cardinals. The technicalities are robust, and so we take for granted one important facts; namely, every cardinal lives inside every larger cardinal. Consequently, the cardinals are indexed by the ordinals. For our purposes we can visualize this as a infinite nesting of boxes full of packing–peanuts, where the smallest box as 0 peanuts, the next largest has 1 peanut, etc:

$$0 \subset 1 \subset 2 \subset \cdots \subset \aleph_0 \subset \aleph_1 \subset \cdots \subset \aleph_\omega \subset \cdots \subset \aleph_{\omega^\omega} \subset \cdots$$

We are now able to state Cantor's definition of God using ordinal and Cardinal arithmetic:

Definition 4 (Cantor)**.** Let Ω^∞ denote the limit of the infinite tower of ordinals

$$0 < 1 < 2 < \cdots < \omega < \omega + 1 < \cdots < \omega^\omega < \cdots$$

Then God, the supreme omnipotent, omnicient, all-encompassing and all–perfect being, is the cardinal $\aleph_{\Omega^\infty}$.

Taking inspiration from Leibniz's observations, Cantor would certainly understand the need to prove some preliminary facts to justify this definition. Notably that Ω^∞ exists in a suitable framework, and that an all–perfect being can be encoded in a cardinal number. Several sources suggest that Cantor developed his own proof of God's existence, yet the author is unable to locate any such credible documentation[10]. Though through study of Cantor's mathematics and theological correspondences with Catholic scholars, we claim that the argument would proceed like something similar to what follows:

Claim 4. *There exists a God*

Proof. Set $\widetilde{\aleph_{\Omega^\infty}}$ to be the disjointified union of all cardinal sets $\aleph_\mathfrak{p}$ as $\mathfrak{p}$ ranges over all ordinals (disjointified just means that even though $\aleph_\mathfrak{p}$ is contained in $\aleph_\mathfrak{q}$ for all ordinals $\mathfrak{q} > \mathfrak{p}$, we consider them to be separate entities inside the ambient object/concept $\widetilde{\aleph_{\Omega^\infty}}$). Philsophically we may interpret $\widetilde{\aleph_{\Omega^\infty}}$ as the space that contains every conceivable thing, feeling, thought, and goodness simultanously. As such, $\widetilde{\aleph_{\Omega^\infty}}$ is either the object $\aleph_{\Omega^\infty}$ (God) that we want, or else $\widetilde{\aleph_{\Omega^\infty}}$ properly contains $\aleph_{\Omega^\infty}$. The latter choice is absurd since God transcends all cardinals (is omniscient). Hence $\widetilde{\aleph_{\Omega^\infty}} = \aleph_{\Omega^\infty}$ and we have constructed an object that contains all of the (in)conceivable infinities.

The final step is to show that $\aleph_{\Omega^\infty}$ must be God. For this it suffices to show that any object or notion X that is all–perfect is necessarily contained in $\aleph_{\Omega^\infty}$. Any such X is the limit (constructed as above) over

[8]I hope the reader can believe me that if person P has an N traceable familiy members then we can continue this assignment and get a 1-1 correspondence

[9]This is $\aleph_0$ with the order described above

[10]Other than an uncited stack–exchange post

all "simple–perfections" (per Leibniz [35, loc. 1518]). Each simple–perfection, in turn, represents a singular concept (which can be indexed by the ordinals, clearly), hence can be represented as the set which contains all smaller (inferior) objects within this concept, and each such set defines a cardinal. It follows that X can be intepreted as the disjoint union over all these simple–perfection cardinals, and so X is a subset of $\aleph_{\Omega^\infty}$. Therefore, any being containing all perfections is contained in $\aleph_{\Omega^\infty}$, and the proof is complete. □

Cantor was unforunately born at the tail-end of the Romantic era. Reactionary socio–politics in Protestant Germany towards Catholic power-structures coupled with the emergence of scientific Positivism put Cantor's mathematical ideas at odds with the mathematical attitude of the times, despite Cantor's Lutheranism. On the other–hand, the slight liberalization of Catholicisim following the mid–19^{th} century Catholic reforms conveniently coincided with some reversions of German theological scholars to more pieist origins, which in turn allowed Cantor to continue working on his mathematical ideas under collaboration with Protestant neologians, Catholic neo–Thomists, and even with Pope Leo XIII [13]. Cantor's "hopes to educate the Roman–Catholic church on the nature of infinity" not only falls in-line with Cantor's belief in God, but it also highlights a major shift in religious attitudes towards rigorous arugments towards the existence of God. Indeed the Cantor–Catholic correspondences stand in stark contrast with Aquinas' response to Anselm, Caetus's response to Descartes, and Pope Innocent's comments about Leibniz, which was simultaneously supportive yet dismissive. In contrast, Leibniz was born in an era and location where the academic community was fervishly enjoying the post–reformation euphoria of the scientific revolution. So reactionary socio–politics was the oppositive to that of Cantor's era. This not only elevated Leibniz in the minds of his contemporaries, but together with 19^{th} century mathematicians' understanding of the significance of Leibniz's mathematical work, Leibniz's theological work was sheilded from criticism by the generation of Positivist scientists contemporary to Cantor. The Romantic era separating Leibniz's time from Cantor's was the major influence behind Cantor's banishment from the German mathematical community and his subsequent collaborations with Catholic theological figures. The philosophical involution induced by the Romantic era meant that traditional Christian theologians would become open to transfinitism while pure scientific thought became more cognizant of Man's finite capacity to comprehend the universe and the divine.

5. Conclusion

Since the end of the dark–ages, ontological arguments for the existence of God have been presented by four key scholarly figures, each one being influenced by – and responding to – the last. While classifying these ontological proofs according to historical context and mathematical/theological significance, one is naturally led to a peculiar dichotomy between the lives and influence of Leibniz and Cantor. Originating in the incompatibility of finite and transfinite knowledge, clerical opposition to the ontological argument remained constant up until the Romantic era, maintaining Man's finite capacity. All while rationalistic natural–philosophers held that Man's ability to comprehend God and the divine was only limited to creativity and ingenuity. The Romantic era was a clear shift in attitude. The author could go on to argue that there is not simply *one* Christian religion; but rather, two fundamentally distinct schools of christian thought. One whose underlying philosophical presupposition is that God (the infinite) cannot be known by Man (the finite), and one that postulates Man's ability to continuously obtain a deeper understanding of nature. Perhaps then the theological question dividing the Christian sect becomes: *is Man finite, or transfinite?*

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