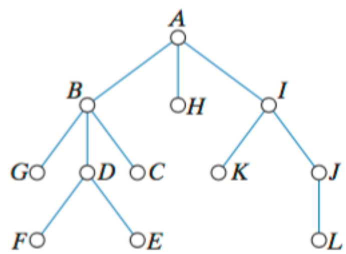
1.1

. For the graph G shown in the figure,



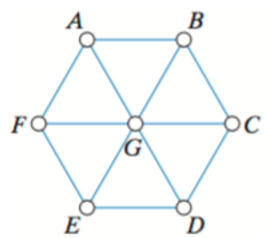
(a) find a 3-coloring of G.

(b) find a 2-coloring of G.

(c) find $\chi(G)$. Explain your answer.

1.2

For the graph G shown in the figure,



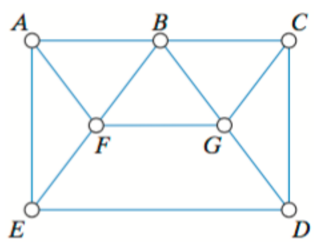
(a) find a 4-coloring of G.

(b) find a 3-coloring of G.

(c) find $\chi(G)$. Explain your answer.

1.3

For the graph G shown in the figure,



(a) find a 5-coloring of G.

(b) find a 4-coloring of G.

(c) find $\chi(G)$. Explain your answer.

2.1 Let K_5 denote the complete graph on 5 vertices. Explain why $\chi(K_5) = 5$.

2.2 Let K_n be the complete graph with n vertices, where n can be any whole number. Argue why $\chi(K_n)$ is exactly equal to n.

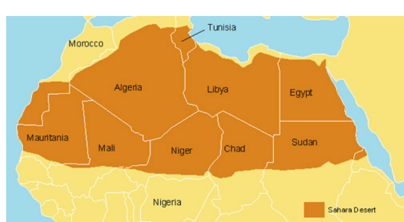
2.3 Let us modify K_n by choosing some edge e, and then removing it completely from the graph. Call the graph K_n with e removed G. What is $\chi(G)$? (Hint: First do the problem for $n = 3$, $n = 4$, and $n = 5$. Note that G is still a complete graph on m vertices - what is m?).

For each of the following maps, constuct the border graph G and determine the chromatic number $\chi(G)$.

3.1

3.2

3.3



Below is a map of the United States. Answer the questions that follow.

4.1 Find 10 states so that the border graph G is $2-$colorable.

4.2 Find 4 states whose border graph cannot be colored with fewer than 4 colors.

4.3 Using the last problem, make an argument for why one cannot color the entire border map of the United States using fewer than 4 colors. What is the chromatic number of the border graph for all lower 48 states?

"Bonus" Problem (Hard):

5. Think back to the Königsberg seven bridge problem. We are city planners who towns hire to build bridges. The mayors of several towns come to you with some prototype plans to build bridges in their towns, but all of the Mayors are worried that their plans do not have solutions to the "bridge problem", meaning you cannot walk across all of the bridges exactly once. The graphs for the town bridges are pictured below (ignore the numbers below). For each picture, decide if you can or cannot solve the "Kónigsberg problem". Justify your answers.