Recall that we say a magic square of order n is *normal* if its entries are exactly the numbers $1, 2, 3, \ldots, n^2$, i.e a normal magic square of order 3 will have the entries $1, 2, 3, 4, 5, 6, 7, 8, 9$.

0: How many normal Magic squares are there of order 1?

1: Does there exist a normal magic square of order 2? If so the find an example. If not then argue why no such square exists.

2: Below is a partially filled in normal magic square of order 3. Complete the square, then try to find as many *different* magic squares of order 3 as possible (Hint: there are 8 in total).

2	7	
	5	
		8

3: Consider the following six normal magic squares:

8	1	6
3	5	7
4	9	2

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

32	29	4	1	24	21
30	31	2	3	22	23
12	9	17	20	28	25
10	11	18	19	26	27
13	16	36	33	5	8
14	15	34	35	6	7

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

64	2	3	61	60	6	7	57
9	55	54	12	13	51	50	16
17	47	46	20	21	43	42	24
40	26	27	37	36	30	31	33
32	34	35	29	28	38	39	25
41	23	22	44	45	19	18	48
49	15	14	52	53	11	10	56
8	58	59	5	4	62	63	1

1. For each square determine its order and the magic number.
2. Do you see a relationship between the order of the square and its magic

number? (Hint: Remember that a normal magic square of order n has entries $1, 2, 3, \ldots, n^2$, and that each of the n rows sum to the same value).

3. Take the order 3 square above and multiply each of its entries by 5. What is the sum of each row, column, and diagonal? (This is what we call an irregular magic square since we dont use the numbers $1, \ldots 9$. The number you computed is called the *irregular magic number*).

4. Suppose we are given a magic square of order n, where n is a whole number. Let's call our square S. Suppose also we have another whole number – let's call it t. Deduce a formula for the irregular magic number of the square obtained by multiplying each entry of S by t.

5. With S, n, and t as in part (4), write down a formula for the irregular magic number for the square obtained by *adding* t to each entry in S.

4: Consider the following partially filled in normal magic square of order 4:

1	15	14	
12		7	9
8	10	11	
	3	2	16

1. Fill in the rest of the square.

2. What is the magic number?

3. Now make a square of order 4 where the entries are the *squares* of the entries above. For instance, the top row of the new square will have entries $1, 225, 196, \ldots$. What is the sum of each column? Each row? Is this square magic?

4. Make a square where the entires are the *cubes* of the entries of the original square, i.e the first row will be $1, 3375, 2744, \ldots$. What is the sum of all numbers appearing on the diagonal lines? Now add all the numbers that are not on a diagonal. What do you notice? Make a proposition and try to prove it.