A couple of weeks ago we discussed area and volume formulas. A few of these formulas can be found in the table below. I recommend that you memorize these for next years' competitions.

Shape	Parameters	Perimeter	Surface Area	Volume
Circle	Radius r	$2\pi r$	πr^2	N/A
Rectangle	Side lengths l & w	$2l + 2w$	$l \times w$	N/A
Triangle	Sides a, b, c	$a + b + c$	$\sqrt{s(s-a)(s-b)(s-c)}$, $s = \frac{a+b+c}{2}$	N/A
Sphere Open Cylinder	Radius r Radius r and height h	N/A $4\pi r$	$4\pi r^2$ $2\pi rh$	$\frac{4}{3}\pi r^3$ N/A
Closed Cylinder	r and h	N/A	$2\pi r^2 + 2\pi rh$	$\pi r^2 h$
Cone	Radius r and side lenght s	N/A	πrs	$\frac{1}{3}\pi r^2 h$
Pyramid	Base length l, base width w, height h	N/A	$lw + l\sqrt{\left(\frac{w}{2}\right)^2 + h^2} + w\sqrt{\left(\frac{l}{2}\right)^2 + h^2}$	$\frac{1}{3}lwh$

The "parameters" that define these shapes are the ***GEOMETRIC*** data of the shape, and things like area and volume are known as *GEOMETRIC* properties. The purpose of our discussion today is to see what kind of math we can still do if we don't specify the parameters of shapes. Afterall, a circle is still a circle even if the radius is unknown; and likewise for squares, pyramids, etc. The properties of shapes without geometric data are called ***TOPOLOGICAL*** properties. **Before completing the exercises below, make sure that you have 4 strips of paper and some tape.**

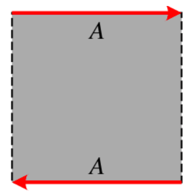
1: Take one strip of paper and draw a straight line down the middle of the long end. Make a cylinder by taping one end to the other without making any half twists.

1. How many *sides* does the cylinder have?
2. How many *borders* does it have?

Using a pair of scissors cut the cylinder along the line you drew.

1. How many pieces remain after you cut the cylinder?
2. What shapes are these pieces? Count the total number of sides and edges.
3. Suppose that for each of the pieces you obtained, you again cut it all the way around down its middle line. Argue how many pieces will remain, what shape the pieces are, and how many sides and borders there will be.

2: Take a second strip of paper, draw a line down its middle, and make a single half-twist to the strip before taping the ends together (as illustrated below). The shape we get is called the *Möbius band*



1. How many *sides* does the Möbius band have? (Hint: The original strip had two sides with two colors, but observe how the two colors "meet" on the Möbius band)
2. How many *borders* does the Möbius band have?

Now cut the Möbius band along the line you drew.

1. How many pieces pieces remain after cutting?
2. hy this happens? (Hint: Think of the line you cut as an edge between two parallel Möbius bands that we glued together)

Cut the resulting shape down the middle line again. What is the result?

3: Draw a line down a third strip of paper and then make a band that has 2 half-twists in it.

1. Guess the resulting number of pieces, sides, and borders if you cut this surface down the middle line.
2. Cut the surface and compare the result with your guess.

4: Suppose you made a surface with n half-twists in it, where n is any positive whole number. How many pieces will the surface separate into if you cut it down the middle line? (Hint: There are two cases to consider, based on the number of sides and borders the original, uncut surface has).