Recall that in *last digit arithmetic* we say that $a \overset{10}{=} b$ if $a - b$ is divisible by 10.

0: For each of $(1) - (6)$, compute each of the terms seprately, and then compare the first item with the second two.

1. $R_{10}(5+3)$, $R_{10}(5)$, $R_{10}(3)$
2. $R_{10}(16+17)$, $R_{10}(16)$, $R_{10}(17)$
3. $R_{10}(23+7)$, $R_{10}(23)$, $R_{10}(7)$
4. $R_{10}(2 \times 4)$, $R_{10}(2)$, $R_{10}(4)$
5. $R_{10}(5 \times 3)$, $R_{10}(5)$, $R_{10}(3)$
6. $R_{10}(11 \times 12)$, $R_{10}(11)$, $R_{10}(12)$

1: As you saw above, if we are given two numbers a and b, then it would appear as if the following formula holds:

$$R_{10}(a+b) \overset{10}{=} R_{10}(a) + R_{10}(b) \tag{1}$$

$$R_{10}(a \times b) \overset{10}{=} R_{10}(a) \times R_{10}(b) \tag{2}$$

Equation 1 means if we take the last digit of the sum $a+b$, then that seems to be equal to the last digit of a added to the last digit of b. Similarly 2 means if we take the last digit of the product $a \times b$, then that is equal to the last digit of a times the last digit of b.

Prove 1 and 2 (Hint: Write $a = 10x + R_{10}(a)$ and $b = 10y + R_{10}(b)$ for some whole number quotients x and y. Now compute $a+b$ and $a \times b$, and then use the definition of $\overset{10}{=}$).

2: Compute $R_{10}(9^{2017})$

3: We now explore last digit arithmetic with respect to different *moduli* (everything up to this point has been done using the modulus 10).

Definition 1. *Fix a whole number $n > 0$. Given numbers a and b, we say that*

$$a \overset{n}{=} b \text{ if } a - b \text{ is divisible by } n$$

Additionally, we denote by $R_n(a)$ the remainder of a after division by n. In other words, there is a quotient q so that $a = q \times n + R_n(a)$.

1. Compute $R_5(16+17)$, $R_5(16)$, and $R_5(17)$.
2. Compute $R_6(13 \times 4)$, $R_6(13)$, and $R_6(4)$.
3. Compute $R_7(2^{2017})$.
4. Prove 1 and Prove 2, replacing R_{10} with R_n and $\overset{10}{=}$ by $\overset{n}{=}$ (Hint: Use $a = q \times n + R_n(a)$ as in problem 2).