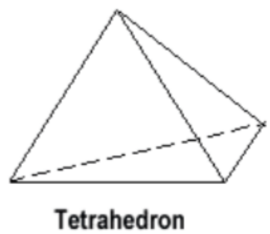
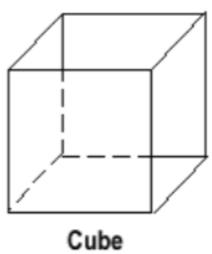
Recall that a regular $n-$gons is a convex shape in the plane with n sides and n angles of equal size.

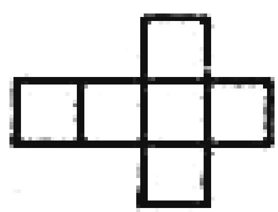
When we studied the regular tessellations of the plane, we saw that there were only three ways to tile a plane with one regular shape; namely with *triangles, squares and hexagons.*

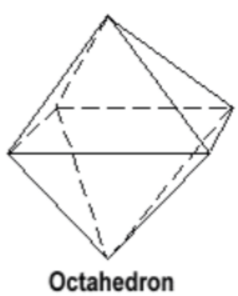
Today we will investigate an extension of this idea by allowing our shapes to live in 3-dimensions instead of just on a 2-dimensional piece of paper.

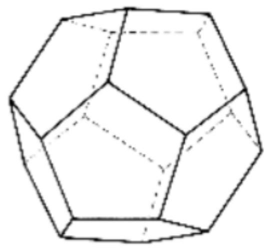
1.1 On the left are 5 Platonic solids. Imagine that each Platonic solid is made of paper. We can "unfold" each solid into a *net*, as shown to the right of the cube. Draw the net for each of the other Platonic solids.

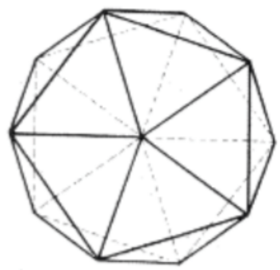












1.2 Do the following:

1. By drawing a small •, mark off the vertex points on each of the Platonic solids on the left hand side of the diagrams above.
2. Mark the corrsponding points on the net you drew to the right.
3. Using colored pencils or the | and || notation, label all of the edges of the solids on the left, and then match them to the corresponding edges on the right.

2.1 Fill in the following table using figures and your answeres to problem **1**. *Key*: V = total # of vertices in the Platonic solid, E = # edges, F = # faces.

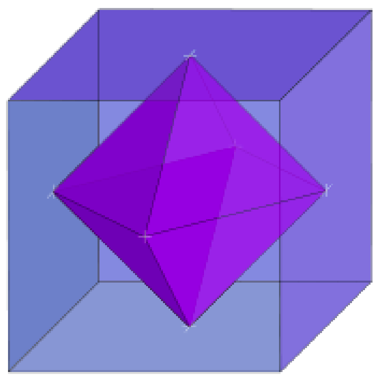
Platonic Solid	V	E	F	V - E + F
Tetrahedron				
Cube				
Octahedron				
Dodecahedron				
Icosahedron				

2.2 Make a proposition above $V - E + F$ using the information in the table from **2.1**.

3. In this problem we will examine an interesting property, known as *duality*, that the Platonic solids possess. The basic idea is that for every Platonic solid P we can construct a new plantonic solid Q.

Let P be a Platonic solid. We construct Q using the following procedure:

1. Draw a vertex in the middle of each face of P.
2. If two faces are adjacent, connect the vertices in their centers with a line.
3. Q is the solid whose edges are the lines drawn in step 2. We call Q the *dual* of P. This procedure is illustrated below in the case when P is the cube.



Do the same for all of the Platonic solids and fill in the table below. Are there any Platonic solids P that are *self-dual*, meaning that the dual solid Q is the same solid as P?

Platonic Solid	*Dual* Platonic Solid
Tetrahedron	
Cube	
Octahedron	
Dodecahedron	
Icosahedron	

4 (Hard): A *polyhedron* is any shape build from polygons. A *convex* polyhedron is one that does not have any *cave* vertices (ask me what this is if you attempt this problem).

Theorem 1 (Euler's formula for polyhedra)**.** *If P is a convex polyhedron whose faces are regular $n-$gons so that around each vertex there are p polygons, then*

$$\frac{1}{n} + \frac{1}{p} - \frac{1}{2} = \frac{1}{E}$$

where $E = \#$ of edged on P.

Use the above theorem to prove that the five solids pictured in problem **1** are the *only* Platonic solids.

We finish this lesson by noting how the Platonic solids fit into the geometric world we have studied so far. Think back to the tilings of the plane by regular triangles, squares, and hexagons.

At each vertex we could have 6 regular triangles, 4 squares, or 3 regular hexagons. We denote these congurations by $T_{triangle}$,T_{square}, and $T_{hexagon}$, respectively.

Let $s = \#$ of sides on a regular polygon, and let $p = \#$ of polygons meeting at one vertex. Then all the geometric objects we have studied fit into the following table:

$p \backslash s$	2	3	4	5	6	7	$\cdots$
2	S	S	S	S	S	S	$\cdots$
3	S	Tetrahedron	Cube	Dodecahedron	$T_{hexagon}$	H	$\cdots$
4	S	Octahedron	T_{square}	H	H	H	$\cdots$
5	S	Icosahedron	H	H	H	H	$\cdots$
6	S	$T_{triangle}$	H	H	H	H	$\cdots$
7	S	H	H	H	H	H	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

You might be wondering what the S and H are indicating. S corresponds to tilings of the *sphere* and the congiuguations with a H are *hyperbolic* geometric objects. We may mention these in the future.