When studying the *Euler characteristic* of the platonic solids, we were essentially trying to solve the equation $F - E + V = 2$ using positive whole numbers for F, E, V. So for instance, the cube is a platonic solid since $(F, E, V) = (6, 8, 12)$ is a solution. When we only care about whole number solutions, equations like $F - E + V = 2$ or $a^2 + b^2 = c^2$ are called *Diophantine equations*. The integers that are solutions to these equations have interesting geometric interpretations.

1. Whole number solutions to $a^2 + b^2 = c^2$ are called *Pythagorean triples*. These correspond to the possible integers that can arise as the side lengths of a right triangle.

   (a) Give an example of POSITIVE whole numbers a, b, and c that make $a^2 + b^2 = c^2$ true.

   (b) Let x, y, and z be the values you found in (a). Now plug the NEGATIVE numbers $-x, -y$, and $-z$ into the original equation: is the equation still true?

   (c) What is the solution to $a^2 + b^2 = c^2$ where $a + b + c$ is SMALLEST?

   (d) What is the solution to $a^2 + b^2 = c^2$ where $a + b + c$ is CLOSEST TO 0?

   (e) (*Difficult*) Write down a set of instructions on how to start with ONE pythagorean triple (x, y, z), do some operations to x, y, and z, and then obtain a NEW pythagorean triple (q, r, s). Can you write instructions on how to get INFINITELY MANY DIFFERENT pythagorean triples only starting with ONE triple? (There are several different answers to this problem).

2. (**Homework**) *Pell's equation* is the Diophantine equation that looks like $X^2 - nY^2 = 1$, where n is a positive whole number that we start with. For example, $X^2 - Y^2 = 1$ is Pell's equation for $n = 1$. The whole number solutions X and Y to Pell's equation correspond to the points with integer coordinates on a hyperbola.

   (a) In this problem we always assme $n = 2$, so Pell's equation is $X^2 - 2Y^2 = 1$. At first, X could theoretically be either odd or even. Argue why X is ALWAYS odd.

   (b) Using (a), show that Y is ALWAYS even.

   (c) Explain why X can never be equal to Y.

   (d) Explain why X is never bigger than 3 or never smaller than -3 (Hint: what happens when $X \geq 4$ or $X \leq -4$?).

   (e) Based on your answers for $(a) - (d)$, what are the 6 solutions to the $n = 2$ Pell equation?